Mirror Surface Reconstruction under an Uncalibrated Camera

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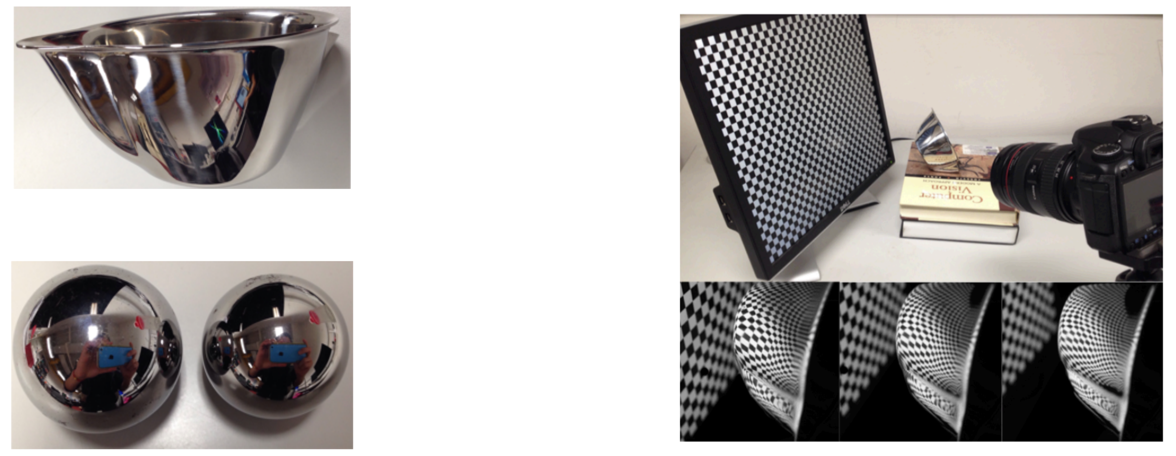
Problem Definition & Contribution

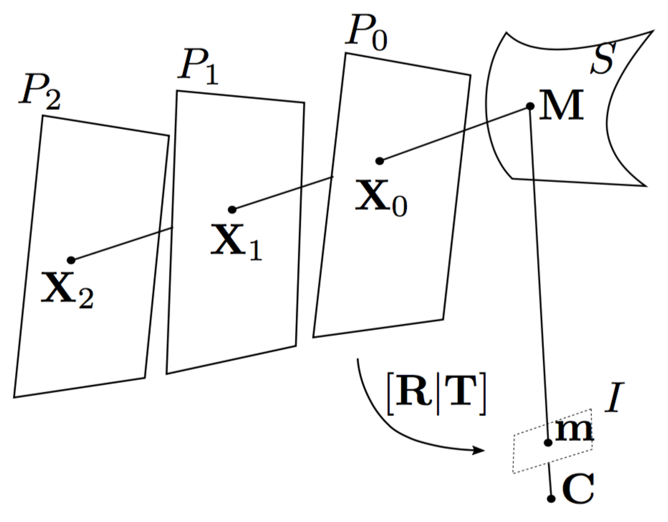
We address the problem of mirror surface reconstruction, and propose a solution based on observing the reflections of a moving reference plane on the mirror surface. Unlike previous approaches which require tedious work to calibrate the camera, our method can recover both the camera intrinsics and extrinsics together with the mirror surface from reflections of the reference plane under at least three unknown distinct poses.



The key contributions of this work are:

- To the best of our knowledge, the first mirror surface reconstruction solution under an unknown motion and an uncalibrated camera.
- A closed-form (linear) solution for estimating the camera projection matrix from reflection correspondences.
- A cross-ratio based nonlinear formulation that allows a robust estimation of the camera projection matrix together with the mirror surface.

Point-Line Correspondences

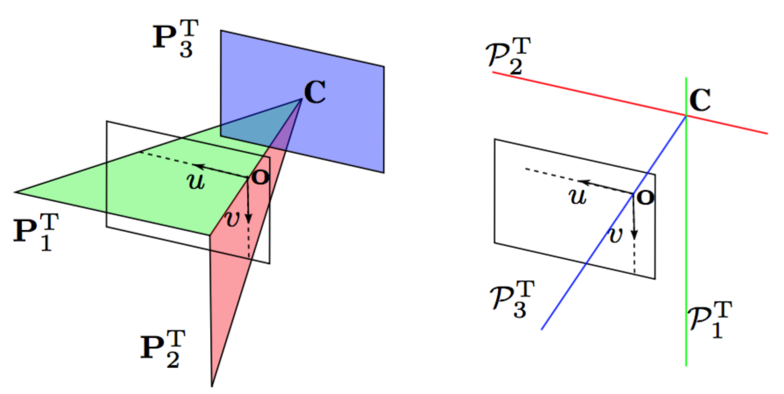


The relative poses of the reference plane under three distinct poses can be estimated using reflection correspondences established across the images[1]. We can then form a 3D Plücker line $\mathcal{L}$ from the reflection correspondences of each observed point x in the image.

Hence, we can obtain a set of 3D space line correspondences $\{\mathcal{L}_1, \ldots, \mathcal{L}_n\}$ for a set of image points $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$.

[1] M. Liu, K.-Y. K. Wong, Z. Dai, and Z. Chen. Pose estimation from reflections for specular surface recovery. In *ICCV*, pages 579–586, 2011.

Line Projection Matrix



Point projection: $\mathbf{x} = \mathbf{P}\mathbf{X}$, $\mathbf{P}$ is a 3 x 4 matrix.

Line projection: $\mathbf{l} = \mathcal{P}\bar{\mathcal{L}}$, $\mathcal{P}$ is a 3 x 6 matrix.

A valid line projection matrix must satisfy

$$\mathcal{P}_i \cdot \bar{\mathcal{P}}_j = 0 \ \forall\, i, j \in \{1, 2, 3\}$$

Closed-form Solution

$$\mathbf{x}^{\mathrm{T}}\mathcal{P}\bar{\mathcal{L}} = 0 \quad \text{<=>} \quad \mathbf{A}\vec{\mathcal{P}} = \mathbf{0}\,, \quad \text{where } \vec{\mathcal{P}} = [\mathcal{P}_1^{\mathrm{T}}\ \mathcal{P}_2^{\mathrm{T}}\ \mathcal{P}_3^{\mathrm{T}}]^{\mathrm{T}} \text{ and } \mathbf{A} = \begin{bmatrix} \mathbf{x}_1^{\mathrm{T}} \otimes \bar{\mathcal{L}}_1^{\mathrm{T}} \\ \vdots \\ \mathbf{x}_n^{\mathrm{T}} \otimes \bar{\mathcal{L}}_n^{\mathrm{T}} \end{bmatrix}$$

($\otimes$ stands for Kronecker product)

$$\underset{\mathcal{P}}{\operatorname{argmin}} \sum_{i=1}^{n} \frac{(\mathbf{x}_i^{\mathrm{T}}\mathcal{P}\bar{\mathcal{L}}_i)^2}{a_i{}^2 + b_i{}^2}$$

, where a_i and b_i are parameters for the 2D line $\mathcal{P}\bar{\mathcal{L}}_i$.

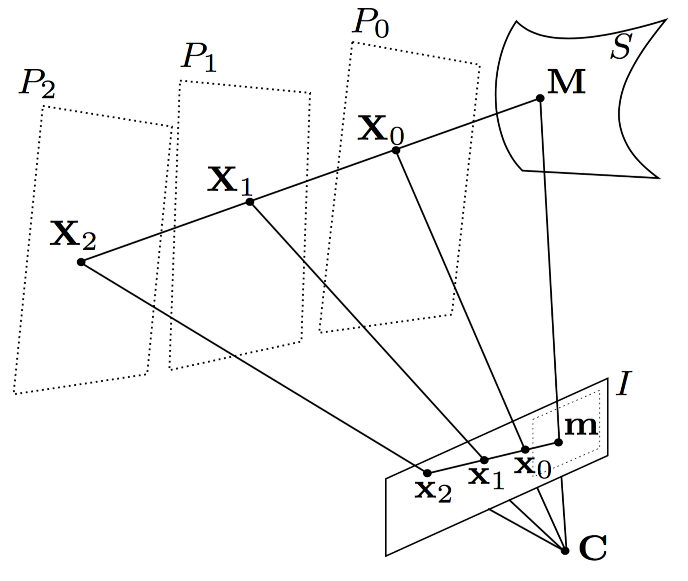
Enforcing $\mathcal{P}_i \cdot \bar{\mathcal{P}}_j = 0 \ \forall\, i, j \in \{1, 2, 3\}$

Given (u_0, v_0), $\mathbf{P} = \mathbf{K}[\mathbf{R}\ \mathbf{T}] = \begin{bmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix}$ <=> $\mathcal{P} = \begin{bmatrix} f_y & 0 & 0 \\ 0 & f_x & 0 \\ 0 & 0 & f_x f_y \end{bmatrix} \mathcal{P}'$

$$\mathbf{A}\vec{\mathcal{P}} = \mathbf{A}\mathbf{D}\vec{\mathcal{P}}' = \mathbf{A}'\vec{\mathcal{P}}' = 0$$

where $\mathbf{D}$ is a 18 x 18 diagonal matrix with $d_{ii} = f_y$ for $i = \{1, \ldots, 6\}$, $d_{ii} = f_x$ for $i = \{7, \ldots, 12\}$, and $d_{ii} = f_x f_y$ for $i = \{13, \ldots, 18\}$.

Cross-ratio Based Formulation



$$\mathbf{M} = \mathbf{X}_2 + s\frac{\overrightarrow{\mathbf{X}_2\mathbf{X}_0}}{|\overline{\mathbf{X}_2\mathbf{X}_0}|}$$

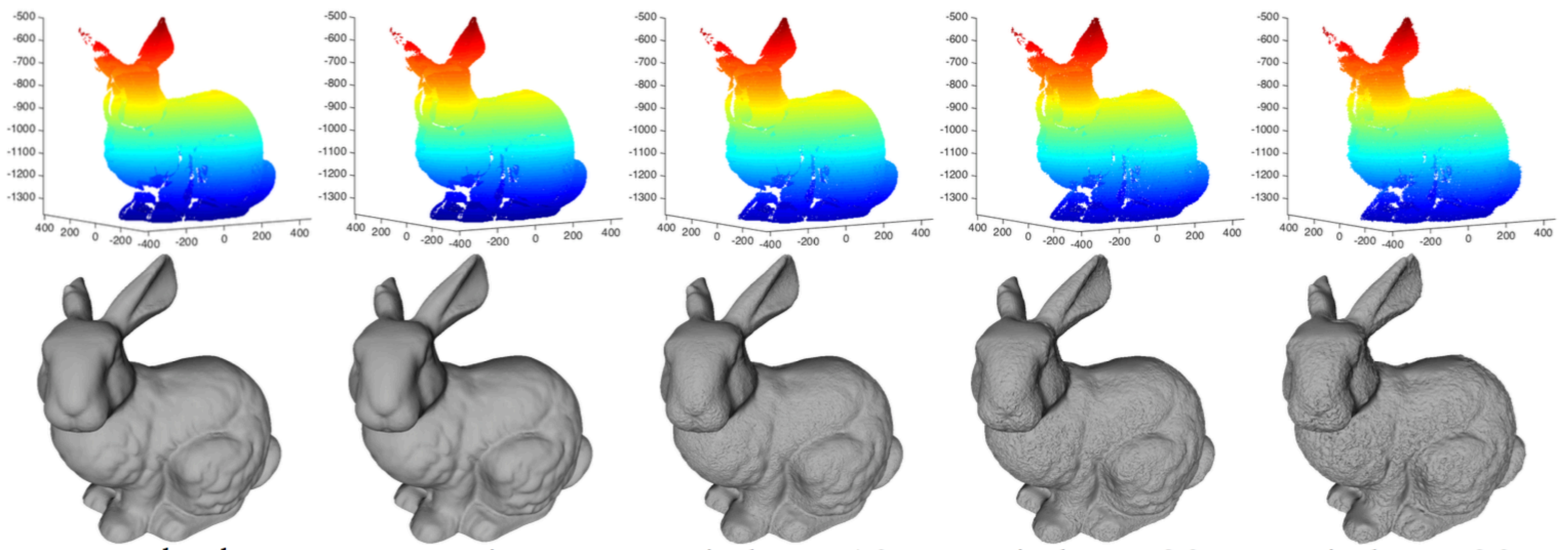
$$\text{CR:} \quad \frac{|\overline{\mathbf{X}_1\mathbf{M}}||\overline{\mathbf{X}_2\mathbf{X}_0}|}{|\overline{\mathbf{X}_1\mathbf{X}_0}||\overline{\mathbf{X}_2\mathbf{M}}|} = \frac{|\overline{\mathbf{x}_1\mathbf{m}}||\overline{\mathbf{x}_2\mathbf{x}_0}|}{|\overline{\mathbf{x}_1\mathbf{x}_0}||\overline{\mathbf{x}_2\mathbf{m}}|}$$

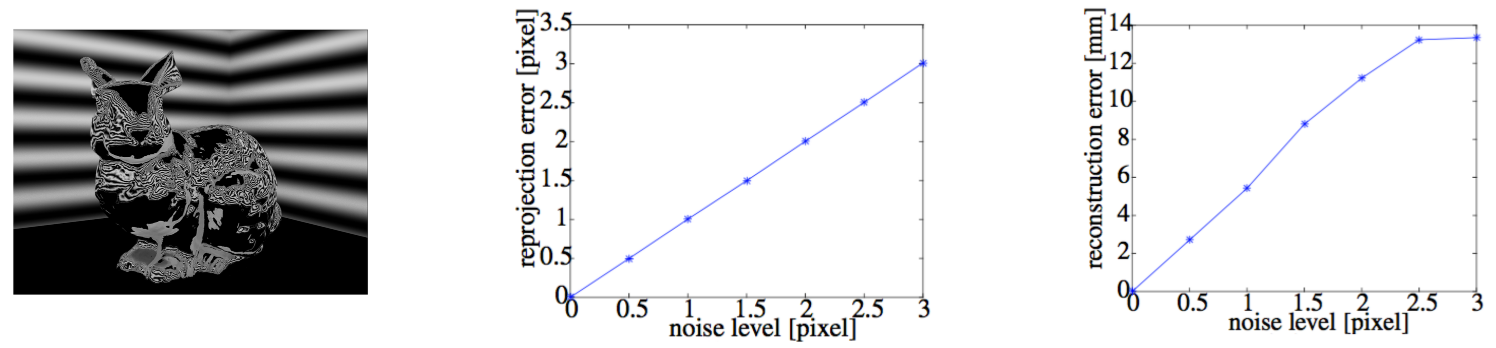
$$s = \frac{|\overline{\mathbf{X}_2\mathbf{X}_1}||\overline{\mathbf{X}_2\mathbf{X}_0}||\overline{\mathbf{x}_1\mathbf{x}_0}||\overline{\mathbf{x}_2\mathbf{m}}|}{|\overline{\mathbf{X}_2\mathbf{X}_0}||\overline{\mathbf{x}_1\mathbf{x}_0}||\overline{\mathbf{x}_2\mathbf{m}}| - |\overline{\mathbf{X}_1\mathbf{X}_0}||\overline{\mathbf{x}_2\mathbf{x}_0}||\overline{\mathbf{x}_1\mathbf{m}}|}$$

$$\underset{\theta}{\operatorname{argmin}} \sum_{i=1}^{n} (\mathbf{m}_i - \mathbf{m}'_i)^2$$

where $\mathbf{m}'_i = \mathbf{P}(\theta)\mathbf{M}_i$, $\theta = [f_x, f_y, u_0, v_0, r_x, r_y, r_z, t_x, t_y, t_z]$

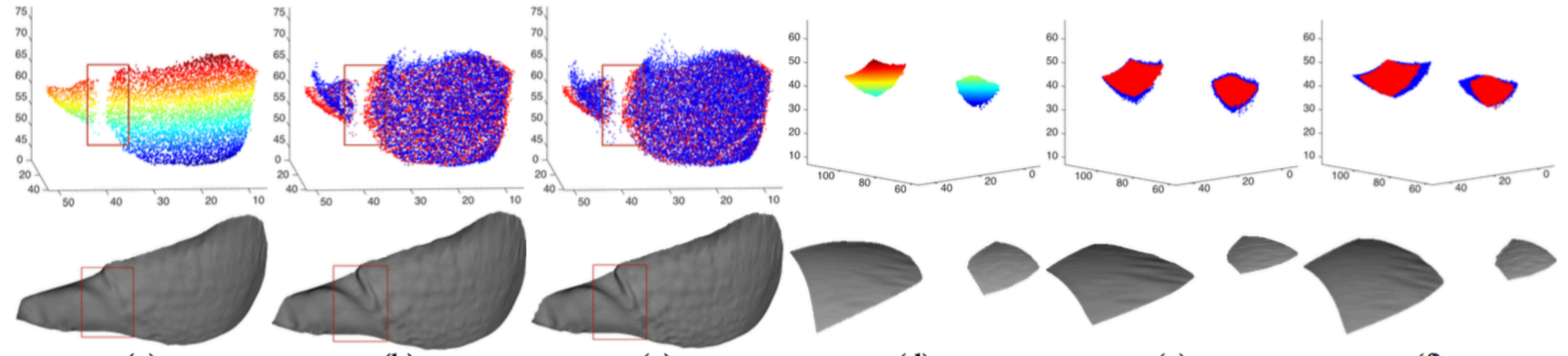
Experimental Results: Synthetic Data





	f_u [pixel]	f_v [pixel]	u_0 [pixel]	v_0 [pixel]	$\mathbf{R}$ [°]	$\mathbf{T}_{deg}$ [°]	$\mathbf{T}_{scale}$ [mm]
$\sigma = 0.5$	0.31(0.02%)	0.31(0.02%)	0.49(0.08%)	0.38(0.08%)	0.03	0.03	0.90(0.05%)
$\sigma = 1.0$	0.22(0.02%)	0.22(0.02%)	0.57(0.09%)	0.63(0.13%)	0.04	0.03	0.93(0.05%)
$\sigma = 1.5$	0.62(0.04%)	0.62(0.04%)	0.63(0.10%)	0.15(0.03%)	0.03	0.03	0.92(0.05%)
$\sigma = 2.0$	2.02(0.14%)	2.02(0.14%)	1.17(0.18%)	0.43(0.09%)	0.06	0.07	2.91(0.16%)
$\sigma = 2.5$	7.22(0.52%)	7.22(0.52%)	5.18(0.81%)	2.03(0.42%)	0.22	0.28	11.24(0.62%)
$\sigma = 3.0$	19.11(1.36%)	19.11(1.36%)	13.11(2.05%)	5.01(1.04%)	0.57	0.72	28.79(1.59%)

Experimental Results: Real Data



(a) (b) (c) (d) (e) (f)

	f_u [pixel]	f_v [pixel]	u_0 [pixel]	v_0 [pixel]	$\mathbf{R}$ [°]	$\mathbf{T}_{deg}$ [°]	$\mathbf{T}_{scale}$ [mm]	S_{rms} [mm]
B_{uc}	36.70(0.63%)	21.99(0.38%)	99.10(5.03%)	100.00(8.13%)	9.12	1.00	19.16(8.23%)	2.55
B_{uu}	101.70(1.75%)	86.90(1.49%)	112.10(5.69%)	113.00(9.19%)	9.86	1.99	17.02(7.34%)	2.71
S_{uc}	63.38(1.09%)	68.01(1.17%)	61.49(3.18%)	42.7(3.47%)	6.67	1.78	33.83(8.96%)	1.78
S_{uu}	81.38(1.40%)	86.02(1.48%)	81.67(4.14%)	56.70(4.61%)	7.17	2.13	37.69(9.98%)	2.03